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Comparison of Shock-Induced Two- and Three-Dimensional Incipient Turbulent Separation

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Introduction

SHOCK waves resulting from sudden compressions in axial corners or in rectangular ducts, interact two-dimensionally with the boundary layer on the compression surface and three-dimensionally (skewed or glancing shock) with the layer on the adjacent surface. The configuration is illustrated in Fig. 1. It is typical of the flow in "two-dimensional" supersonic diffusers.

The purpose of this Note is to present available quantitative data for conditions of incipient separation of turbulent boundary layers due to two-dimensional and skewed shock wave interactions and thus show that separation will occur much earlier for the latter than for the former types of interaction. This information is designed to bring together what is presently known about two and three-dimensional shock wave-turbulent interactions; however, much more data is needed for in-depth knowledge and for defining characteristics of separation.

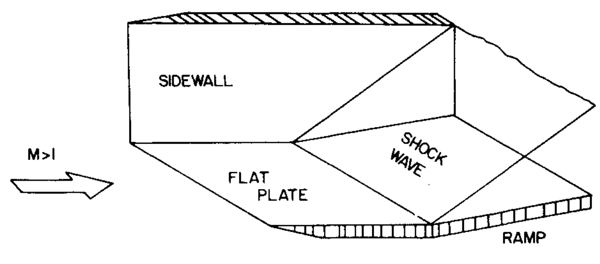

Fig. 1

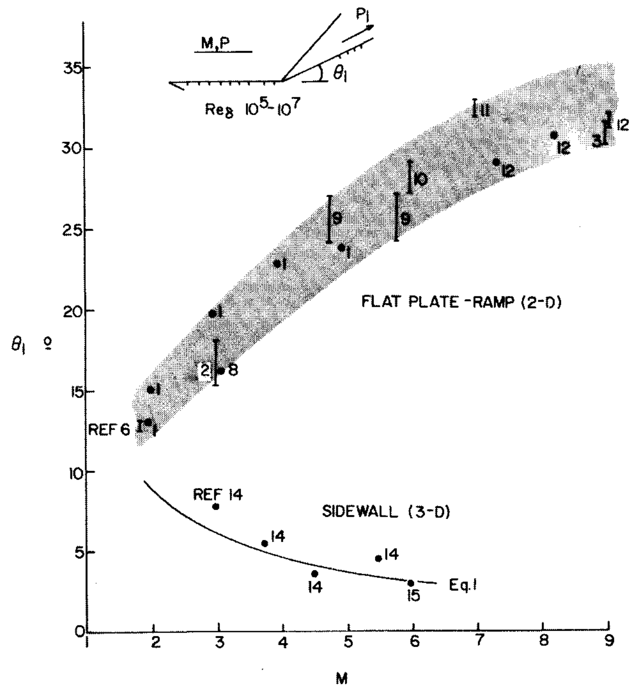

Fig. 2 Incipient separation angle.

Incipient Separation Data

In order to avoid the possibility of transitional effects, only data corresponding to $Re_\delta > 10^5$ are considered. According to experimental evidence, the compression angle θ_i for two-dimensional incipient turbulent boundary-layer separation first decreases, then increases slowly[1,2] with increasing Re_δ; decreasing wall temperature also results in some increase in θ_i.[3] However, these variations are considerably smaller than for the laminar case.[4]

For incipient turbulent boundary-layer separation due to a skewed shock wave, the present author[5] has shown that existing data up to $M = 3.5$ correlate according to

$$M\theta_i = 0.3 \text{ (radians)} \tag{1}$$

Corresponding to which the pressure rise across the shock wave is $P_i/p = 1.50$ independent of M.

Incipient separation data[1–3,6–15] for both two-dimensional and skewed shock wave-turbulent boundary-layer interactions are shown in Fig. 2. The numerals indicate references. The vertical lines indicate data over a wide range of Reynolds numbers and/or limits of accuracy, if known. Included in Fig. 2 for the three-dimensional case are new data points from Neumann and Token[14] up to $M = 5.5$ and one from Law[15] which tend to confirm the trend of the correlation. Additional data on three-dimensional interactions at $M = 5.9$ by Goldberg[16] is not included because the incipient separation angle could not be determined with sufficient accuracy.

The pressure rise for incipient separation corresponding to Fig. 2 is shown in Fig. 3, along with two empirical correlations for the two dimensional case, adequate for rough estimates, in which no account is taken of the effect of Reynolds number and wall temperature variations as these are not large to a first order. The correlations are:

$$C_{p_i} = 0.43 \quad \text{or} \quad p_i/p = 1 + 0.3M^2 \qquad M \lesssim 4.5 \tag{2}$$

and

$$p_i/p = 0.17M^{2.5} \qquad M \gtrsim 4.5 \tag{3}$$

Received September 10, 1974.
Index categories: Boundary Layers and Convective Heat Transfer—Turbulent: Supersonic and Hypersonic Flows.

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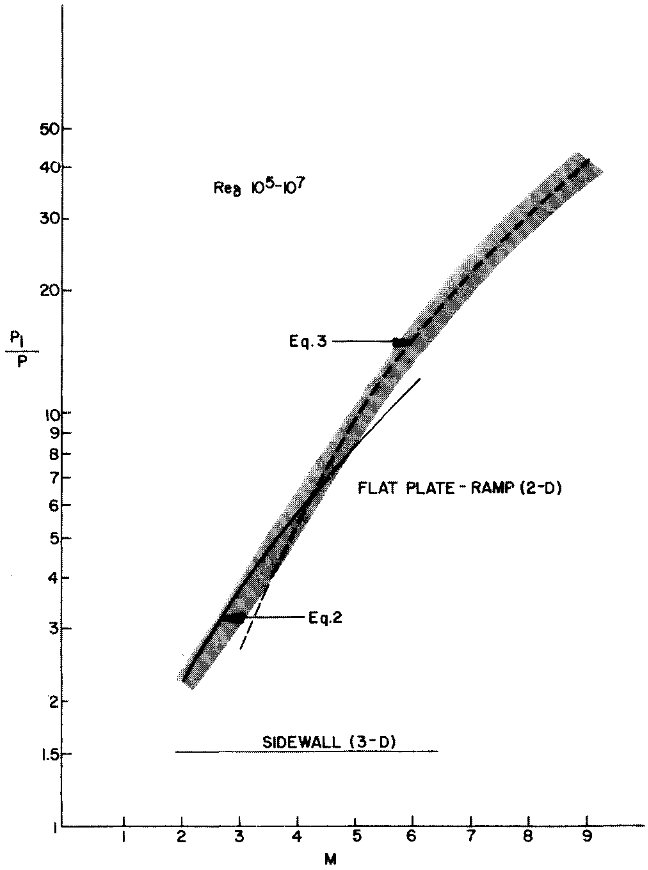

Fig. 3 Incipient separation pressure rise.

Conclusions

Although two-dimensional data cover a wide band of incipient separation angles θ_i, it is clear that the values are considerably higher than for skewed shock wave interactions, and the gap widens with increasing Mach number. Thus, it is the skewed shock wave interaction with the sidewall turbulent boundary layer in rectangular diffusers or inlets (see Fig. 1) that first leads to separation and possible flow breakdown for compression angles (or pressure rises) which may be well below the incipient values for the two-dimensional case.

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Interferometric Technique for Measuring Mixing of a Buoyant Plume

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WHEN investigating the three-dimensional and unstable character of buoyant plumes, it is important to use a measurement technique which does not disturb the flowfield. Interferometry provides a simple means of making detailed concentration measurements under conditions which have a range of applicability. This Note concerns itself with a special case, namely, the isothermal, isobaric plume. A simple development follows which shows how interferometry can be used to measure buoyant plume mixing of a light gas into a heavy background gas, all at constant temperature and pressure. Finally, an example of the results is given in the form of reduced data of a vortex-like plume.

The standard equation for fringe shift is:

$$S = 1/\lambda \int_0^L (n - n_\infty) ds \tag{1}$$

where S = fringe number, λ = wave length of light in vacuum, L = integration path length, n = index of refraction, s = path length along the light ray, and subscript ∞ refers to reference conditions.

An interferogram yields the fringe number S, and the solution of the integral equation provides the index field, n. An equation for n can be developed in terms of the species mass fraction C_i and Gladstone-Dale constants K_i as

Received August 8, 1974; revision received September 23, 1974.

Index categories: Jets, Wakes and Viscid-Inviscid Flow Interactions; Shock Waves and Detonations.

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